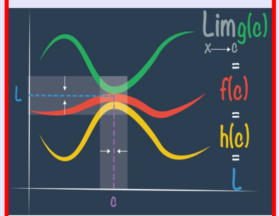
Calculus I

Lecture 5



Feb 19-8:47 AM

Find K such that

$$f(x) = \begin{cases} Kx^2 & \text{if } x < -1 \\ Kx + 4 & \text{if } x \geq -1 \end{cases}$$

is cont. at $x = -1$.

→ we need to show

$$\lim_{x \to -1} f(x) = f(-1)$$

$= -K + 4$

1) $f(-1) = K(-1) + 4 = -K + 4$

2) $\lim_{x \to -1} f(x) \Rightarrow \lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x)$

$K(-1)^2 = K(-1) + 4$

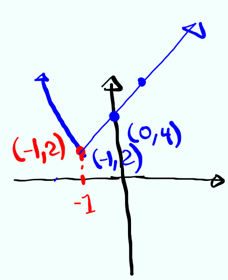
$K = -K + 4$

$K + K = 4$

$2K = 4$

$\boxed{K = 2}$

$$f(x) = \begin{cases} 2x^2 & \text{if } x < -1 \\ 2x + 4 & \text{if } x \geq -1 \end{cases}$$



Jan 12-8:05 AM

Find K such that

$$f(x) = \begin{cases} K + \sin x & \text{if } x < 0 \\ 2K - \cos x & \text{if } x \geq 0 \end{cases}$$

Cont. at $x = 0$.

$\lim_{x \to 0} f(x) = f(0)$

$= 2K - \cos 0$

$= 2K - 1$

$\lim_{x \to 0^-} f(x) = K + \cancel{\sin 0}^{\to 0} = K$

$\lim_{x \to 0^+} f(x) = 2K - \cancel{\cos 0}^{\to 1} = 2K - 1$

$\Rightarrow 2K - 1 = K$

$2K - K = 1$

$\boxed{K = 1}$

Jan 12-8:13 AM

Show the equation $2\sin x + 2x - 3 = 0$ has a solution in the interval $[0, 1]$.

Hint: Use I.V.T.

Intermediate Value Theorem.

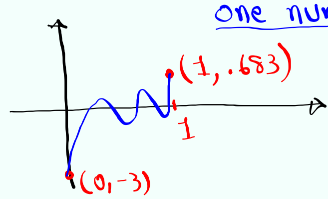
$f(x) = 2\sin x + 2x - 3$

Continuous everywhere

$f(0) = 2\sin 0 + 2(0) - 3 = 0 + 0 - 3 = \boxed{-3}$

$f(1) = 2\sin 1 + 2(1) - 3 = \boxed{.683}$

by I.V.T., $f(x) = 0$ for at least one number in $(0, 1)$.



Jan 12-8:18 AM

For $\varepsilon>0$, find a $\delta>0$ such that

$$\lim_{x\to 1} \frac{4x+2}{3} = 2.$$

$f(x) = \frac{4x+2}{3}$

$a = 1$

$L = 2$ ✓

For every $\varepsilon>0$, there is a $\delta>0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x-a|<\delta$

$\left|\frac{4x+2}{3} - 2\right| < \varepsilon$ 〃 $|x-1|<\delta$

Multiply by 3

$|4x+2-6| < 3\varepsilon$

$|4x-4| < 3\varepsilon$

$|4(x-1)| < 3\varepsilon$

Divide by 4

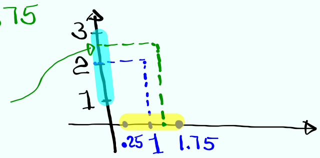
$|x-1| < \frac{3\varepsilon}{4}$

choose $\boxed{\delta = \frac{3\varepsilon}{4}}$

If $\varepsilon = 1$, $\delta = \frac{3(1)}{4} = .75$

Pick $x = 1.5$

$f(1.5) = \frac{4(1.5)+2}{3} = \frac{8}{3} = 2.\bar{6}$



Jan 12-8:25 AM

For $\varepsilon = .25$, find $\delta>0$ such that

$$\lim_{x\to 2} (x^2-4x+5) = 1.$$

$f(x) = x^2-4x+5$

$a = 2$

$L = 1$ ✓

For every $\varepsilon>0$, there is a $\delta>0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x-a|<\delta$

$|x^2-4x+5-1| < \varepsilon$ 〃 $|x-2|<\delta$

$|x^2-4x+4| < \varepsilon$ 〃 $|x-2|<\delta$

Factor

$|(x-2)^2| < \varepsilon$

Take square root of both sides

$|x-2| < \sqrt{\varepsilon}$

Pick $\delta = \sqrt{\varepsilon}$

$\delta = \sqrt{.25}$

$\boxed{\delta = .5}$

Jan 12-8:35 AM

Prove using ε & δ such that

$\lim_{x \to \frac{1}{2}} \frac{1}{x} = 2.$

$f(x) = \frac{1}{x}$

$a = \frac{1}{2}$

$L = 2$ ✓

For every $\varepsilon > 0$, there is a $\delta > 0$ Such that $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$\left|\frac{1}{x} - 2\right| < \varepsilon$ whenever $\left|x - \frac{1}{2}\right| < \delta$

$\left|\frac{1 - 2x}{x}\right| < \varepsilon$

$\left|\frac{-2x + 1}{x}\right| < \varepsilon$

$\left|\frac{-2\left(x - \frac{1}{2}\right)}{x}\right| < \varepsilon$

$\frac{|-2|\left|x - \frac{1}{2}\right|}{|x|} < \varepsilon$

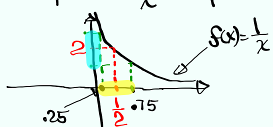
$\frac{2}{|x|}\left|x - \frac{1}{2}\right| < \varepsilon$

Bound Keep

if $\frac{2}{|x|} < C$

$C\left|x - \frac{1}{2}\right| < \varepsilon$

$\left|x - \frac{1}{2}\right| < \frac{\varepsilon}{C}$



Can we say $\delta \le \frac{1}{4}$?

Pick $\delta = \min\left\{ \ , \frac{\varepsilon}{C}\right\}$

Jan 12-8:45 AM

So $\delta \le \frac{1}{4}$

$\left|x - \frac{1}{2}\right| < \frac{1}{4}$

$-\frac{1}{4} < x - \frac{1}{2} < \frac{1}{4}$

Add $\frac{1}{2}$

$-\frac{1}{4} + \frac{1}{2} < x < \frac{1}{4} + \frac{1}{2}$

$\frac{1}{4}$

$.25 < x < .75$ $\qquad \frac{3}{4}$

$4 > \frac{1}{x} > \frac{4}{3}$

$\frac{4}{3} < \frac{1}{x} < 4$

$\frac{1}{|x|} < 4$

$\frac{2}{|x|} < 8$

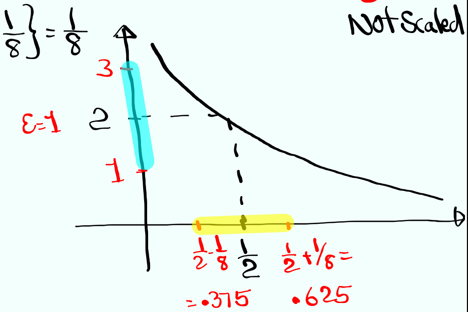
C

$\delta = \min\left\{\frac{1}{4}, \frac{\varepsilon}{8}\right\}$

If $\varepsilon = 1$

$\delta = \min\left\{\frac{1}{4}, \frac{1}{8}\right\} = \frac{1}{8}$

Not Scaled



Jan 12-8:58 AM

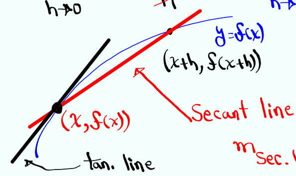
Given $f(x) = x^2 + 6x + 5$, evaluate

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 + 6(x+h) + 5 - x^2 - 6x - 5}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{+6x} + 6h \cancel{+5} \cancel{-x^2} \cancel{-6x} \cancel{-5}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x + h + 6)}{\cancel{h}} = \lim_{h\to 0} (2x + \overset{\to 0}{h} + 6)$$

$$= \boxed{2x + 6}$$



$$\boxed{\begin{aligned} f(x) &= x^2 + 6x + 5 \\ f'(x) &= 2x + 6 \end{aligned}}$$

$$m_{\text{sec. line}} = \frac{f(x+h) - f(x)}{h}$$

when $h \to 0$

we have a tan. line $\quad m_{\text{tan. line}} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

The expression that gives the general form of the slope of the tan. line is called the first derivative of $f(x)$.

Function $f(x)$

First derivative $\quad \boxed{f'(x) \quad \text{F - Prime of } x}$

Jan 12-9:07 AM

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$ if the limit exists.

$$f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}$$ if the limit exists.

$f(x) = x^3$, $f'(a)$

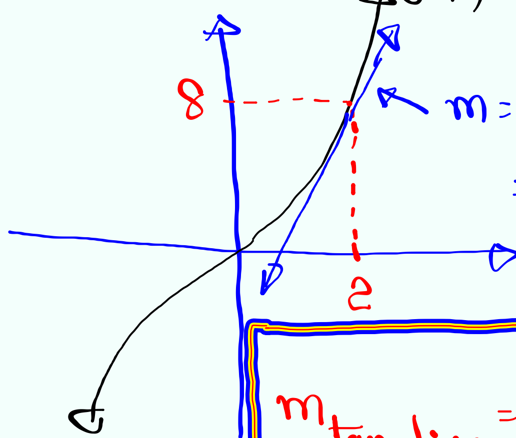
$$f'(a) = \lim_{x\to a} \frac{x^3 - a^3}{x - a} = \lim_{x\to a} \frac{\cancel{(x-a)}(x^2 + xa + a^2)}{\cancel{x-a}}$$

$$= \lim_{x\to a} (x^2 + xa + a^2)$$

$$= a^2 + aa + a^2 = \boxed{3a^2}$$

Jan 12-9:21 AM

$f(x) = x^3 \qquad a = 2$



$m = f'(a)$

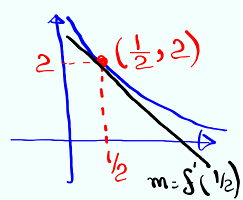
$= 3a^2 = 3(2)^2 = \boxed{12}$

$$\boxed{m_{\text{tan. line}} = f'(\text{tan. Point})}$$

Jan 12-9:26 AM

Given $f(x) = \frac{1}{x}$

1) find $f\left(\frac{1}{2}\right) = \frac{1}{\frac{1}{2}} = \boxed{2}$



2) find $f'(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

LCD $= (x+h)x$

$$= \lim_{h \to 0} \frac{x - (x+h)}{h(x+h)x} = \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}(x+h)x}$$

$$= \lim_{h \to 0} \frac{-1}{(x+h)x} = \frac{-1}{x^2}$$

$m = f'\left(\frac{1}{2}\right)$

$= \frac{-1}{\left(\frac{1}{2}\right)^2} = \boxed{-4}$

Eqn of tan. line at $x = \frac{1}{2}$

$y - y_1 = m(x - x_1)$

$y - 2 = -4\left(x - \frac{1}{2}\right) \rightarrow \boxed{y = -4x + 4}$

Jan 12-9:29 AM

class QZ 7

open notes

$f(x) = x^2 - 8x$

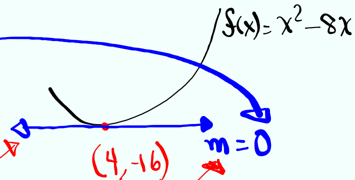
Use $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ to find

$f'(x)$, then evaluate $f'(4)$.

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^2 - 8(x+h) - x^2 + 8x}{h} = \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{-8x} - 8h \cancel{-x^2} \cancel{+8x}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x+h-8)}{\cancel{h}} = \lim_{h \to 0}(2x + \cancel{h}^{\to 0} - 8) = \boxed{2x-8}$$

$f'(4) = 2(4) - 8 = \boxed{0}$



horizontal tan. line

$f(4) = 4^2 - 8(4) = \boxed{-16}$

Jan 12-9:36 AM

$f(x) = \dfrac{x-2}{x+1}$

1) find $f(0) = \dfrac{0-2}{0+1} = \dfrac{-2}{1} = \boxed{-2}$

2) find $f'(x)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{x+h-2}{x+h+1} - \frac{x-2}{x+1}}{h}$$

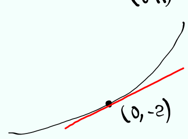
LCD $= (x+h+1)(x+1)$

$$= \lim_{h\to 0} \frac{(x+h-2)(x+1) - (x-2)(x+h+1)}{h(x+h+1)(x+1)}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} \cancel{+x} \cancel{+hx} + h \cancel{-2x} \cancel{-2} \cancel{-x^2} \cancel{-xh} \cancel{-x} \cancel{+2x} + 2h \cancel{+2}}{h(x+h+1)(x+1)}$$

$$= \lim_{h\to 0} \frac{3\cancel{h}}{\cancel{h}(x + \cancel{h}^{\to 0} + 1)(x+1)} = \boxed{\frac{3}{(x+1)^2}}$$

3) find $f'(0) = \dfrac{3}{(0+1)^2} = \boxed{3}$



$m = f'(0) = 3$

Eqn of tan. line

$y - y_1 = m(x - x_1)$

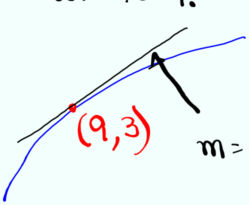
$y - (-2) = 3(x - 0)$

$y + 2 = 3x$

$\boxed{y = 3x - 2}$

Jan 12-10:09 AM

Find equation of the tan. line to the graph of $f(x)=\sqrt{x}$ at $x=9$.



$$f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to0}\frac{\sqrt{x+h}-\sqrt{x}}{h}=\lim_{h\to0}\frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})}$$

$$=\lim_{h\to0}\frac{\cancel{x}+h\cancel{}-\cancel{x}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})}=\lim_{h\to0}\frac{1}{\sqrt{x+\cancel{h}^{\to0}}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$$

$$m=f'(9)=\frac{1}{2\sqrt{9}}=\frac{1}{6}$$

$$y-y_1=m(x-x_1)$$

$$y-3=\frac{1}{6}(x-9)$$

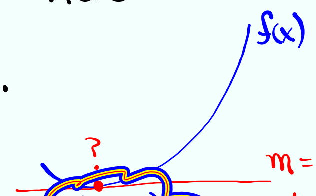
$$y=\frac{1}{6}x-\frac{9}{6}+3$$

$$\boxed{y=\frac{1}{6}x+\frac{3}{2}}$$

Jan 12-10:20 AM

Find all points on the graph of $f(x)=x^3-3x$ where there are horizontal tan. lines.

$f(1)=1^3-3(1)$
$=-2$
$f(-1)=(-1)^3-3(-1)$
$=2$



$m=0$

$f'(x)=0$

$3x^2-3=0$

$x^2=1$

$x=\pm1$

(1,-2)
(-1,2)

$$f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to0}\frac{(x+h)^3-3(x+h)-x^3+3x}{h}$$

$$=\lim_{h\to0}\frac{\cancel{x^3}+3x^2h+3xh^2+h^3\cancel{-3x}-3h\cancel{-x^3}\cancel{+3x}}{h}$$

$$=\lim_{h\to0}\frac{\cancel{h}(3x^2+\cancel{3xh}^{\to0}+\cancel{h^2}^{\to0}-3)}{\cancel{h}}=\boxed{3x^2-3}$$

Jan 12-10:29 AM

find $f'(x)$ for any quadratic function.

$f(x) = ax^2 + bx + C,\ a \neq 0$

$$f'(x) = \lim_{h\to 0} \frac{\overbrace{a(x+h)^2 + b(x+h) + C}^{f(x+h)} \ \overbrace{-ax^2 - bx - C}^{-f(x)}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{ax^2} + 2axh + ah^2 \cancel{+bx} + bh \cancel{+C} \cancel{-ax^2} \cancel{-bx} \cancel{-C}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2ax + \underbrace{ah}_{\to 0} + b)}{\cancel{h}} = \boxed{2ax + b}$$

$$f(x) = \underset{a}{5}x^2 \underset{b}{-8}x + \underset{c}{4}$$

$$f'(x) = 2\cdot 5x - 8 = \boxed{10x - 8}$$

Jan 12-10:43 AM

find $f'(x)$ for $f(x) = x^{-2}$.

Recall $x^{-n} = \dfrac{1}{x^n}$

$f(x) = \dfrac{1}{x^2}$

$$f'(x) = \lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

LCD $= (x+h)^2 x^2$

$$= \lim_{h\to 0} \frac{x^2 - \overbrace{(x+h)^2}^{(x^2+2xh+h^2)}}{h(x+h)^2 x^2} = \lim_{h\to 0} \frac{\cancel{x^2} \cancel{-x^2} - 2xh - h^2}{h(x+h)^2 x^2}$$

$$= \lim_{h\to 0} \frac{-\cancel{h}(+2x + h)}{\cancel{h}(x+h)^2 x^2} = \lim_{h\to 0} \frac{-(2x + \underbrace{h}_{\to 0})}{(x+\underbrace{h}_{0})^2 x^2}$$

$$= \frac{-2\cancel{x}}{x^2 \cdot \cancel{x^2}} = \boxed{\frac{-2}{x^3}}$$

Jan 12-10:52 AM

Find $f'(4)$ for $f(x) = x^{-1/2}$.

$$f(x) = \frac{1}{x^{1/2}} = \frac{1}{\sqrt{x}}$$

$$f'(x) = \lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$$ LCD: $\sqrt{x+h}\,\sqrt{x}$

$$= \lim_{h\to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x+h}\,\sqrt{x}}$$

$$= \lim_{h\to 0} \frac{(\sqrt{x} - \sqrt{x+h})(\sqrt{x} + \sqrt{x+h})}{h\sqrt{x+h}\,\sqrt{x}\,(\sqrt{x} + \sqrt{x+h})}$$

$$= \lim_{h\to 0} \frac{\cancel{x} - (\cancel{x} + \cancel{h})}{\cancel{h}\sqrt{x+h}\,\sqrt{x}\,(\sqrt{x} + \sqrt{x+h})}$$

$$= \lim_{h\to 0} \frac{-1}{\sqrt{x+\cancel{h}^{\,0}}\,\sqrt{x}\,(\sqrt{x} + \sqrt{x+\cancel{h}^{\,0}})}$$

$$= \frac{-1}{\sqrt{x}\sqrt{x}(\sqrt{x}+\sqrt{x})} = \boxed{\frac{-1}{2x\sqrt{x}}}$$

$$f'(4) = \frac{-1}{\sqrt{4}\sqrt{4}(\sqrt{4}+\sqrt{4})} = \frac{-1}{2\cdot 2(2+2)} = \boxed{\frac{-1}{16}}$$

Jan 12-11:01 AM

Given $\lim_{x\to 2} f(x) = 5$ and $\lim_{x\to 2} g(x) = -1$

Find $\lim_{x\to 2} \left[3f(x) - g(x)\right]^2$

$$= \left[\lim_{x\to 2} (3f(x) - g(x))\right]^2$$

$$= \left[\lim_{x\to 2} 3f(x) - \lim_{x\to 2} g(x)\right]^2$$

$$= \left[3\lim_{x\to 2} f(x) - \lim_{x\to 2} g(x)\right]^2 = \left[3\cdot 5 - (-1)\right]^2$$

$$= (15+1)^2 = 16^2 = \boxed{256}$$

Jan 12-11:13 AM

Given $\begin{cases} 4\lim_{x\to5} f(x) - 3\lim_{x\to5} g(x) = 8 \\ 3\lim_{x\to5} f(x) + \lim_{x\to5} g(x) = 5 \end{cases}$

Find $\lim_{x\to5} f(x)$ & $\lim_{x\to5} g(x)$.

Let $A = \lim_{x\to5} f(x)$, $B = \lim_{x\to5} g(x)$

$$3\begin{cases} 4A - 3B = 8 \\ 3A + B = 5 \end{cases} \Rightarrow \begin{cases} 4A - 3B = 8 \\ 9A + 3B = 15 \end{cases}$$

$$13A = 23$$

$$A = \frac{23}{13}$$

$$\boxed{\lim_{x\to5} f(x) = \frac{23}{13}}$$

$$\begin{matrix} -3 \\ 4 \end{matrix}\begin{cases} 4A - 3B = 8 \\ 3A + B = 5 \end{cases}$$

$$\begin{cases} -12A + 9B = -24 \\ 12A + 4B = 20 \end{cases}$$

$$13B = -4$$

$$B = \frac{-4}{13}$$

$$\boxed{\lim_{x\to5} g(x) = \frac{-4}{13}}$$

Jan 12-11:17 AM

Evaluate $\lim_{x\to a} \frac{\sqrt{x} - \sqrt{a}}{x^2 - a^2}$, $\boxed{a>0}$

$$= \frac{\sqrt{a} - \sqrt{a}}{a^2 - a^2} = \frac{0}{0} \quad \text{I.F.}$$

$$= \lim_{x\to a} \frac{\overbrace{(\sqrt{x} - \sqrt{a})}^{A - B}\overbrace{(\sqrt{x} + \sqrt{a})}^{A + B}}{(x^2 - a^2)(\sqrt{x} + \sqrt{a})} = \lim_{x\to a} \frac{\overbrace{\cancel{x - a}}^{A^2 - B^2}}{\cancel{(x-a)}(x+a)(\sqrt{x}+\sqrt{a})}$$

$$= \frac{1}{(a+a)(\sqrt{a}+\sqrt{a})} = \frac{1}{2a \cdot 2\sqrt{a}} = \frac{1}{4a\sqrt{a}}$$

Jan 12-11:27 AM

For $\varepsilon = .2$, find $0 < \boxed{\delta \le 1}$ such that

$$\lim_{x \to 1} (2x^2 + 3x) = 5$$

$f(x) = 2x^2 + 3x$

$a = 1$

$L = 5$ ✓

For every $\varepsilon > 0$, there is a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|2x^2 + 3x - 5| < \varepsilon$ whenever $|x - 1| < \delta$

$|(2x+5)(x-1)| < \varepsilon$ 〃 $|x-1| < \delta$

$\underbrace{|2x+5|}_{\text{Bound}} \underbrace{|x-1|}_{\text{Keep}} < \varepsilon$

if $|2x+5| < C$, then $|x-1| < \frac{\varepsilon}{C}$

If $\delta \le 1$, $|x - 1| < 1$

$-1 < x - 1 < 1$

$+1$

$0 < x < 2$

Multiply by 2

$0 < 2x < 4$

Add 5

$5 < 2x + 5 < 9$

$|2x + 5| < 9$ C

$\delta = \min\left\{1, \frac{\varepsilon}{9}\right\}$

For $\varepsilon = .2$

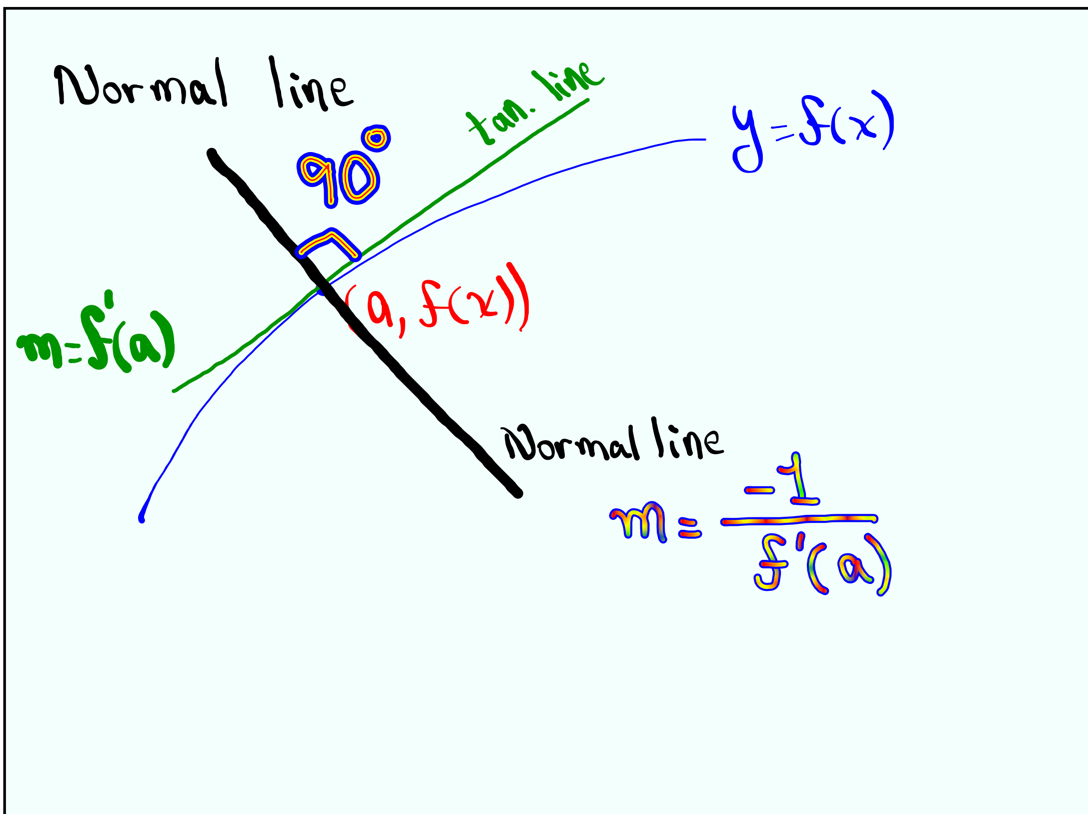
$\delta = \min\left\{1, \frac{.2}{9}\right\}$

$= \min\left\{1, \frac{2}{90}\right\}$

$= \min\left\{1, \frac{1}{45}\right\}$

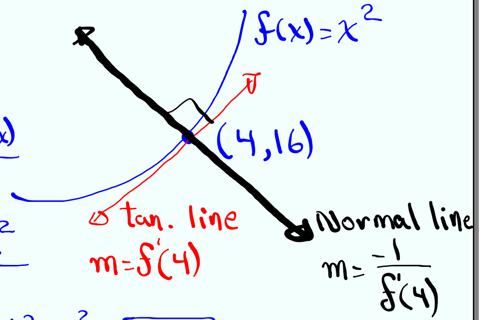
$\boxed{\text{choose } \delta = \frac{1}{45}}$

Jan 12-11:36 AM



Jan 12-11:47 AM

Find slope of the tan. line and the normal line to the graph of $f(x)=x^2$ at $x=4$.



$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}$$

$$=\lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h}=\boxed{2x}$$

$$m_{tan.\,line}=f'(4)=2(4)=\boxed{8}$$

$$m_{Normal\,line}=\frac{-1}{f'(4)}=\boxed{\frac{-1}{8}}$$

Jan 12-11:50 AM

Open notes:

Class QZ 6

Warning: Notation Matters

Evaluate
$$\lim_{x\to 0}\frac{1-\sqrt{1-x^2}}{x}=\frac{1-\sqrt{1-0^2}}{0}=\frac{1-1}{0}=\frac{0}{0}\ \text{I.F.}$$

$A^2 - B^2$

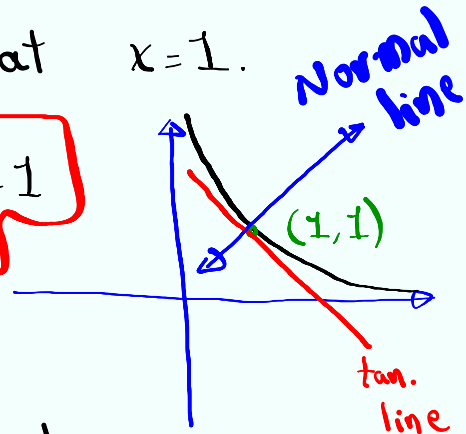
$$=\lim_{x\to 0}\frac{(1-\sqrt{1-x^2})(1+\sqrt{1-x^2})}{x(1+\sqrt{1-x^2})}=\lim_{x\to 0}\frac{1-(1-x^2)}{x(1+\sqrt{1-x^2})}$$

$$=\lim_{x\to 0}\frac{\cancel{1}-\cancel{1}+x^{\cancel{2}}}{\cancel{x}(1+\sqrt{1-x^2})}=\lim_{x\to 0}\frac{x}{1+\sqrt{1-x^2}}$$

$$=\frac{0}{1+\sqrt{1-0^2}}=\frac{0}{2}=\boxed{0}\ \checkmark$$

Jan 8-11:52 AM

Find eqn of the normal line to the graph of $f(x) = \frac{1}{x}$ at $x = 1$.



$$\boxed{m_{\text{Normal line}} = \frac{-1}{f'(1)} = \frac{-1}{\frac{-1}{1^2}} = 1}$$

$f(x) = \frac{1}{x}$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \cdots = \frac{-1}{x^2}$$

Tan. line $y - 1 = -1(x - 1)$

$$\boxed{y = -x + 2}$$

$y - y_1 = m(x - x_1)$

$y - 1 = 1(x - 1)$

$$\boxed{y = x}$$

Jan 12-11:56 AM